MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

DEM5038 – ENGINEERING MATHEMATICS 3

(Diploma in Electronics Engineering)

8 MARCH 2019
3.00 p.m. – 5.00 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **5** pages excluding the cover page.
2. Answer **ALL** questions. All necessary working steps must be shown.
3. Write all your answers in the answer booklet provided.

Please answer ALL questions and show the necessary working. Total mark is equal to 100.

Question 1 (25 marks)

a) Find the solution to the homogeneous differential equation:

$y''+25y=0 \quad y(0)=0; \quad y(\frac{\pi}{10})=1$ [9 marks]

b) Find the solution to the non-homogeneous differential equation:

$y''-2y'=e^{2x} \quad y(0)=0; \quad y'(0)=1$ [16 marks]

Question 2 (25 marks)

a) Given one period of a periodic function as $f(x)=\begin{cases}-1, & -3<x<0 \\ 1, & 0<x<3\end{cases}$

Find the Fourier series for the above function. [16 marks]

b) Find the Half-range Sine series for $f(x)=\begin{cases}x+\pi; & -\pi<x<0 \\ x-\pi, & 0<x<\pi\end{cases};$

$f(x+2\pi)=f(x).$ [9 marks]

Continued...

Question 3 (25 marks)

a) Determine the Laplace transform of the following functions.

i. $3e^{4t} - te^{-2t} + 5$ [2 marks]

ii. $\int_0^t 3\cos 2x dx$ [2 marks]

iii. $f(t) = \begin{cases} 2, & 0 < t < 3 \\ 0, & t \geq 3 \end{cases}$ [4 marks]

b) Find the inverse Laplace transform for the following functions.

i. $\frac{1}{s+2} + \frac{2}{s^2+4}$ [2 marks]

ii. $\frac{4}{(s+3)^2 - 4}$ [2 marks]

c) Find the solution to the following differential equation by using Laplace transform method.

$$y'' - 2y' - 3y = 3e^{2t} \qquad y(0) = 0,\ y'(0) = 1$$ [13 marks]

Continued...

Question 4 (25 marks)

a) According to Chemical Engineering Progress in 1990, 30% of all pipework failures in chemical plants are caused by operator error. Using the binomial distribution function, find the probability that in a random sample of 20 pipework failures,

i. at least 10 are due to operator error. [2 marks]

ii. not more than 4 are due to operator error. [2 marks]

b) The resistance of resistors is normally distributed with mean $150\,\Omega$ and standard deviation of $5\,\Omega$. What percentage of the resistors will have resistance between $148\,\Omega$ and $152\,\Omega$? [2 marks]

c) An average IQ score of school students in a city is 100. The IQ scores are normally distributed. In a recent survey, a random sample of 30 students IQ scores in a school have a mean score of 112 with a standard deviation of 15. Test at 5% significance level if the school students have higher IQ scores?

[7 marks]

d) A study of the amount of rainfall and the quantity of air pollution removed produced the following data:

Table 1

Daily Rainfall x (0.01cm)	**Particulate Removed y** ($\mu g/m^3$)
4.3	126
4.5	121
5.9	116
5.6	118
6.1	114
5.2	118
3.8	132
2.1	141
7.5	108

i. Find the estimate regression line of y on x, $\hat{y} = a + bx$. [10 marks]

ii. Estimate the amount of particulate removed when the daily rainfall is $x = 4.8$ units. [2 marks]

End of Page.

Appendix

Quadratic formula $$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Annihilator for special functions

Annihilator	
D^n $(D-a)^n$ $\left(D^2 - 2aD + (a^2 + b^2)\right)^n$	$1, x, x^2, x^3, x^4, \ldots\ldots\ldots, x^{n-1}$ $e^{ax}, xe^{ax}, x^2e^{ax}, x^3e^{ax}, x^4e^{ax}, \ldots\ldots\ldots, x^{n-1}e^{ax}$ $e^{ax}\cos bx, xe^{ax}\cos bx, x^2e^{ax}\cos bx, x^3e^{ax}\cos bx, \ldots\ldots\ldots, x^{n-1}e^{ax}\cos bx$ and $e^{ax}\sin bx, xe^{ax}\sin bx, x^2e^{ax}\sin bx, x^3e^{ax}\sin bx, \ldots\ldots\ldots, x^{n-1}e^{ax}\sin bx$

Fourier Series

$$F(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}xdx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi}{L}xdx$$

Integration Formula

$$\int \sin axdx = -\frac{1}{a}\cos ax$$

$$\int \cos axdx = \frac{1}{a}\sin ax$$

$$\int x\sin axdx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos axdx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2 \sin axdx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2 \cos axdx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

Linear Regression and Correlation

$$\hat{y} = a + bx$$

$$SS_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n} \qquad SS_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \qquad SS_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$b = \frac{SS_{xy}}{SS_{xx}} \qquad a = \bar{y} - b\bar{x} = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}}$$

Transformation of some functions

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$, s>0
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$ $n = 1,2,3,\ldots$
e^{at}	$\frac{1}{s-a}$, s>a
$t\,e^{at}$	$\frac{1}{(s-a)^2}$
y' y''	$sY(s) - y(0)$ $s^2Y(s) - sy(0) - y'(0)$

$f(t)$	$L\{f(t)\}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
$(1)u(t-a)$	$\frac{e^{-sa}}{s}$
$(t-a)u(t-a)$	$\frac{e^{-sa}}{s^2}$

Second Shift Theorem

$$L\{f(t-a)u(t-a)\} = e^{-as}L\{f(t)\}$$
$$L\{f(t)u(t-a)\} = e^{-as}L\{f(t+a)\}$$

Binomial Probabilities $\binom{n}{x}p^x q^{n-x}$

Binomial Formulae when using Cambridge Statistical Table

Key Formulas (if $p \le 0.5$)

1. $P(X = r) = B(r) - B(r-1)$
2. $P(X \ge r) = 1 - B(r-1)$
3. $P(X > r) = 1 - B(r)$
4. $P(a \le X \le b) = B(b) - B(a-1)$
5. $P(X \le r) = B(r)$

Key Formulas (if $p > 0.5$)

1. $P(X = r) = P(Y = n-r) = B(n-r) - B(n-r-1)$
2. $P(X \ge r) = P(Y \le n-r) = B(n-r)$
3. $P(X \le r) = P(Y \ge n-r) = 1 - B(n-r-1)$
4. $P(a \le X \le b) = P(n-b \le Y \le n-a)$
   $= B(n-a) - B(n-b-1)$

Normal Distribution $Z = \dfrac{X-\mu}{\sigma}$

Hypothesis Testing $S_x = \dfrac{s}{\sqrt{n}}$ $\quad Z = \dfrac{\overline{x}-\mu}{S_x}$